

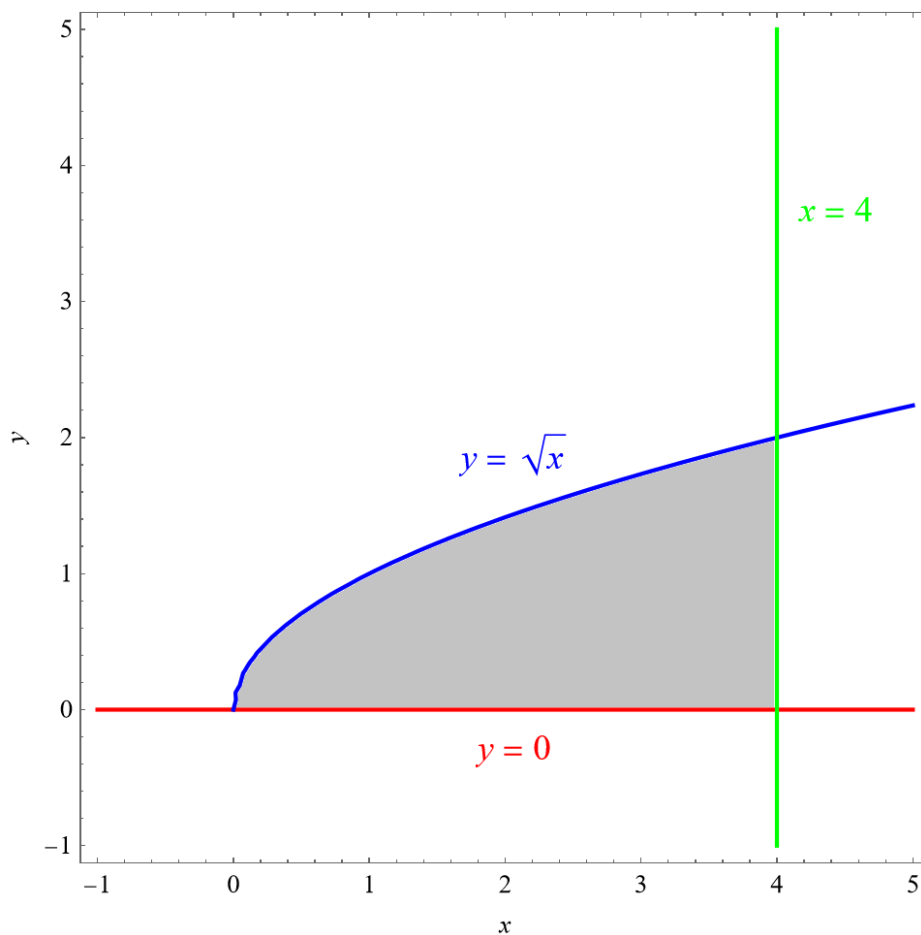
Exercise 45

Sketch the region enclosed by the given curves and calculate its area.

$$y = \sqrt{x}, \quad y = 0, \quad x = 4$$

Solution

Start by drawing the given curves in the xy -plane and shading the area they enclose.



The shaded area is calculated by integrating the height ($\sqrt{x} - 0$) from the lowest value of x to the highest value of x that it occupies.

$$\begin{aligned} \text{Area} &= \int_{x_{\min}}^{x_{\max}} \text{Height } dx = \int_0^4 \sqrt{x} \, dx \\ &= \int_0^4 x^{1/2} \, dx = \left. \frac{x^{3/2}}{\frac{3}{2}} \right|_0^4 = \frac{2}{3}(4^{3/2} - 0^{3/2}) = \frac{2}{3}(8) = \frac{16}{3} \end{aligned}$$